



Bulletin de la Sabix

Société des amis de la Bibliothèque et de l'Histoire de
l'École polytechnique

44 | 2009

Gabriel Lamé (1795-1870) : Les pérégrinations d'un
ingénieur au XIX^e siècle

Symmetry in the physics of Lamé and his contemporaries

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Édition électronique

URL : <http://journals.openedition.org/sabix/684>

DOI : 10.4000/sabix.684

ISSN : 2114-2130

Éditeur

Société des amis de la bibliothèque et de l'histoire de l'École polytechnique (SABIX)

Édition imprimée

Date de publication : 1 octobre 2009

Pagination : 95 - 100

ISBN : ISSN N° 2114-2130

ISSN : 0989-30-59

Référence électronique

Shaul Katzir, « Symmetry in the physics of Lamé and his contemporaries », *Bulletin de la Sabix* [En ligne], 44 | 2009, mis en ligne le 22 mai 2011, consulté le 08 septembre 2020. URL : <http://journals.openedition.org/sabix/684> ; DOI : <https://doi.org/10.4000/sabix.684>

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Introduction

- 1 Symmetry and symmetry breaking are central concepts in contemporary physics. They are employed from the level of the universe as a whole to that of the elementary particles. These considerations have special significance in theories of condense matter (e.g. theory of superconductivity) and in the standard model of high-energy physics. The use of these concepts in the physical sciences goes back to the nineteenth century, while the modern concept of symmetry itself did not originate in physics, but in the geometrical study of crystals. From the 1830s it was a well-defined concept explicitly connected to physics. Nevertheless the relation between symmetry and natural phenomena had not been formulated clearly until the 1880s, and in a general manner not until 1894 when it was defined by Pierre Curie. Meanwhile symmetry had been applied in many fields without a general well-defined rule. Thus, the concept of symmetry in physics was developed with its application. Its history is an interesting example of the development of concepts in science¹.
- 2 In his paper « Sur la symétrie dans les phénomènes physiques » of 1894 Curie analysed the symmetries that characterize physical magnitudes and phenomena and their interrelations :

La symétrie caractéristique d'un phénomène est la symétrie maxima compatible avec l'existence du phénomène.
Un phénomène peut exister dans un milieu qui possède sa symétrie caractéristique ou celle d'un des intergroupes de sa symétrie caractéristique.
- 3 He further suggested two rules for the relations between the symmetry of the causes and that of the effects. Together these rules make up « Curie's principle » :

Lorsque certaines causes produisent certains effets, les éléments de symétrie des causes doivent se retrouver dans les effets produits.

Lorsque certains effets révèlent une certaine dissymétrie, cette dissymétrie doit se retrouver dans les causes qui lui ont donné naissance.

- 4 As will become evident from the following, Lamé and his contemporaries applied consideration of symmetry in a much restricted way than suggested by Curie, and did not suggest explicit discussion of its application. Still, Lamé belonged to those who related symmetry to physical properties. My interest here is in the application of symmetry above an intuitive application of the principle of sufficient reason, e.g. where the physical situation was symmetric, as in the theories of central forces, physicists often presumed that the results would be so too. In the more sophisticated use of symmetry, one considered particular symmetries of specific material media and physical magnitudes and related them to the phenomena.
- 5 In his course on heat conductivity, whose published version appeared in 1861, Gabriel Lamé presented one of the early employment of symmetry in analyzing physical phenomena. His application was innovative, as its details differed from previous uses of symmetry. Yet it was far from revolutionary, as it was grounded in earlier works. It appeared in print almost thirty years after the first mathematical application of symmetry in physics. That despite this Lamé can still be considered as one of the early users of explicit symmetry considerations in physics points at the slow adoption and development of the concept during the nineteenth century. Indeed, the gradual employment and development of these concepts is a central characteristic of their history, (see table 1, which goes beyond the period discussed in this paper). Almost eighty years passed between Haüy's definition of structural symmetry in the modern sense and Curie's full definition of a general symmetry principle for the physical sciences.
- 6 This article focuses on Lamé's uses of symmetry consideration within its context. It starts with a survey of the application of considerations of symmetry that preceded his. This is an essential background for recognising Lamé's sources, for the assessment of his contribution, and the place of symmetry in his thought. The reasons for earlier applications of symmetry and its role in previous theories help suggesting Lamé's reasons to employ such considerations. The answer turns out to be connected to his understanding of mathematical physics. In other words, Lamé's use of symmetry was connected to his philosophy and methodology of science. These are, therefore, described here. His positivistic view of science called for a theory without any hypothesis about the constitution of matter. Symmetry was well suited for such a method, as it is indifferent to questions about the structure of matter. This effort to eliminate unnecessary hypotheses was shared by others early users of consideration of symmetry in physics. However, most of them did not share Lamé's optimism in the ability of such a road to lead to a definitive theory. Although both positions can be termed positivistic, they divided on this important philosophical issue.

Symmetry in physics before Lamé

- 7 In the application of symmetry to physics one can identify two central traditions one « molecular » and the other « phenomenological » (or descriptive). The former is associated with French scientists connected with René-Just Haüy and his disciples ; the latter is associated with German researchers, connected with Franz Neumann and his

disciples. « Interdisciplinary » research of crystal that linked their structure to physical, chemical and even biological phenomena characterized the « French school. » Haüy established structural symmetry in its modern sense, i.e., to put it anachronistically, as a recurrence of the same pattern by a mathematical operation. In 1815 he put forward the basic assumption of this school. According to him the symmetry is determined by the molecular structure of the crystal. That structure determines also the physical properties of the substance. Thus the symmetry and the physical behaviour are connected through the structure. However, Haüy did not employ this connection².

- 8 Gabriel Delafosse, his student, maintained Haüy's view about the primacy of the molecular structure but added to it an influence from a German school of crystallography. The latter 'dynamical' school emphasised mathematical symmetry as a way to avoid material assumptions. In 1840, Delafosse accepted the view, first expressed by that school, according to which physical effects display the symmetry of the crystal form. From that he concluded that the symmetry and hence the molecular structure of some crystals is more complicated than assumed from their outer form alone. The underlying notion that symmetry governs the physical behaviour of crystals, even if indirectly, became common at the time. This notion was supported by experimental evidence regarding cleavage, elasticity, optics, and electricity. However, no experiment had been carried out to test directly the dependence of phenomena on symmetry until the experiments of Hureau de Senarmont. In 1847 and 1850 Senarmont showed experimentally that heat and electric conductivity are subject to the symmetry of the crystal form³. Lamé would later apply the concept of symmetry in a theory of heat conductivity that explains, among others, Senarmont's results.
- 9 Considerations of symmetry were central to Louis Pasteur's work on optical isomerism from 1848: the existence of crystals constituted of molecules of the same chemical structure except for being mirror images of one another. Pasteur was a student of Delafosse, and like his teacher was interested in the relations between the structure and physical properties of crystals, a connection that contributed to his discovery of optical isomerism. After the discovery, Pasteur linked the new property to the symmetry, or more precisely the asymmetry, of crystalline molecules. Asymmetric activity of amyl alcohol was the key for his conjecture that this substance is a product of a living agent. That was the key for his 1857 celebrated experimental identification of a new living agent of fermentation, responsible for the production of amyl alcohol. His view that every fermentation is caused by organic germs originated in that study. His reasoning was based on two hypotheses: first that asymmetry must originate in asymmetry and second that living organisms are the only agents that can produce asymmetry. Considerations of symmetry, thus, played an essential role in the discovery of the organic agents of fermentation⁴. Yet, Pasteur's accomplishments and his hypotheses did not play a role in Lamé's work.
- 10 Across the Rhine, Neumann employed considerations of symmetry as a way to avoid questions of ontology. Neumann adopted methods and concepts of the above-mentioned German crystallographic school. However, while the «dynamic» school used symmetry to reject materialistic molecules, Neumann applied it to maintain an agnostic view about the nature of crystals and forces free from any ontological commitment. Therefore, he connected directly symmetry to the physical properties, disregarding question of ultimate source of the phenomena. This led him in 1832, and

in an explicit form in 1834, to apply for the first time considerations of symmetry to elasticity and the related theory of optics. Until then considerations of symmetry were employed only to the crystalline form, and not to physical properties. More specifically he assumed that the physical magnitudes in symmetric positions (and only in such positions) are equal. In this case a function that expresses the dependence of the elastic effect on the direction have equal values. Since the elastic coefficients are functions of this function he showed how symmetry could be used to reduce their number in different crystals classes. Neumann continued to develop his ideas in university courses, which would be published by his students only in 1885⁵. Lamé, therefore could not have benefited from Neumann's (or his disciples') more elaborated treatment of the subject.

Lamé's application of symmetry

- 11 Crystallography and the physical properties of crystals were important both in the French and the German traditions, as these were virtually the only subjects in which one applied considerations of symmetry. This should not come as a surprise since considerations of symmetry become interesting in case of restricted symmetry. This was the situation in crystals. Implicitly a complete symmetry was often assumed to lead to equal values in all directions. But this did not lead to a discussion of the relation between physical phenomena and symmetry. Lamé's own course on elasticity is a good example, as symmetry is mentioned there only in passim and only in relations to mathematics. Since at the elasticity course Lamé did not examine cases of partial symmetry, e.g. crystals with different behaviour along different axes, he had no need for physical considerations of symmetry. In that course, Lamé referred to full symmetry where magnitudes have the same value in any direction, and to symmetry of mathematical equations, i. e. the interchangeability of two variables in a system of equations. The latter, is only accidentally connected to structural symmetry⁶.
- 12 Another example for the casual use of full symmetry can be found in Fourier's celebrated theory of heat conductivity. Without much discussion of symmetry, in that theory one assumes that the temperature would be the same in two points that are equidistant from the same source of heat. Symmetry started to be interesting in some of the cases that Lamé examined in his 1861 *Leçons sur la théorie analytique de la chaleur*, namely in the extension of the theory to non-isotropic substances, where the heat is not evenly conducted in all directions. Lamé's course was in many respects an exposition of Fourier's *théorie analytique de la chaleur* and its elaborations by other authors. In discussion non-isotropic conductivity, Lamé followed and elaborated the theory of his friend and colleague Jean-Marie Duhamel. The latter developed a theory for non isotropic heating already in 1828 and extended it with specific reference to crystalline matter twenty years later, following the experimental findings of Senarmont. Duhamel did not employ the concept of symmetry in his publications. Lamé, on the other hand, applied symmetry in developing the theory to crystals of particular restricted symmetry. These cases did not receive a similar explicit treatment by Duhamel. Lamé also reformulated a central assumption of Duhamel's theory about the existence of a particular convenient system of coordinate as « the symmetric equality » incorrectly attributing the concept to the latter (p. xviii)⁷. Such 'creative

reading' is well-known in the development of scientific thinking, where one often projects one's own ideas in exploiting the works of others.

- 13 Lamé's use of symmetry was connected to his view of mathematical physics. According to him, the aim of his work was

d'établir la Théorie analytique de la Chaleur, sans partir d'aucun hypothétique relatif à la constitution intérieure des milieux solides, sans présupposer les lois de l'échange calorifique, ou du rayonnement particulaire, sans adopter aucune restriction pour les variations de la conductibilité autour d'un même point. Je suis de plus en plus convaincu qu'en évitant de cette manière toute idée préconçue sur les lois naturelles, la Physique mathématique, aidée par l'expérience et par l'observation, ne tardera pas à découvrir ces lois mêmes. (p. v)

- 14 It is important to note that while Lamé did not allow hypothetical assumptions about the constitution of matter and the nature of heat he showed much confidence in the ability of the scientific research to reach the true laws regarding those questions in the foreseeable future. In that Lamé revealed a positivistic « Comtean » approach, quite different from the much more pessimistic positivistic view according to which one could never gain a secure knowledge about these questions, or even from the view of many scientists, like Franz Neumann, that the structure of matter and similar questions would not be solved in the foreseeable future. This can explain why in this book Lamé did assume the existence of molecules, if not specific assumptions about them ; unlike other scientists he did not regard the existence of molecules as hypothetical. Moreover, following what he regarded as a safe « non hypothetical » road, he even reached conclusions about their structure. According to Lamé, such conclusions about the structure of matter were one of the important aims of the analytical theory of heat.

- 15 Lamé expressed a clear optimism about the future of mathematical physics :

Lorsqu'une branche de la Physique mathématique est ainsi parvenue à écarter tout principe douteux, toute hypothèse restrictive, elle entre réellement dans une phase nouvelle. Et cette phase paraît définitive, car la série historique, et en même temps rationnelle, des progrès accomplis, signale une tendance constante vers l'indépendance de toute loi préconçue. (p. vi)

- 16 Fourier's hypothesis of isotropic conductivity is such a kind of restrictive hypothesis. Its abolishment in Duhamel's theory of non-isotropic conductivity and further in Lamé's own course advances that theory towards its definitive phase (p. ix). Symmetry considerations provided a way to treat specific classes of crystals while avoiding specific assumptions about their molecular structure. They also provided a general and thereby elegant way to discuss mathematical equalities. Lamé's preference to use them might also be connected to his research on crystal geometry, a subject in which symmetry played an important role. He himself had used mathematical notion of symmetry when he published on the subject at the beginning of his career⁸.
- 17 In unfolding his theory Lamé referred to symmetry on a par with other structural properties like orthogonality and oblique coordinates (his term). He used both to posit equalities between theoretical entities that characterize the conduction, like the axes of the ellipsoid that determines the conductivity and the cosines of the planes to that ellipsoid. Both ensure that the equations for heat conductivity would have equal values in those places. For example, in « cas du prisme oblique symétrique » the two symmetric axes of the ellipsoid that characterize heat conduction are equal (p. 50-52).

In some cases Lamé applied the equations to crystals according to their crystalline classification, in others according to their symmetry. In all cases Lamé assumed symmetry of the equations, not of the matter. He did not compare physical magnitudes in symmetrical positions, as was done by the members of Neumann's school, but equate mathematical coefficients. Still the difference is subtle and the mathematical method similar to that used in Neumann's published derivation. Unlike Neumann, he did not connect particular crystal classes to specific symmetries in a clear way. Taking all this together, one can conclude that symmetry is far from an organizing principle in Lamé's work.

- 18 Symmetry considerations had been used for the mathematical theory of heat conductivity already a decade earlier by George Gabriel Stokes, another mathematical physicist. In 1851, Stokes, the new Lucasian Professor of Mathematics at Cambridge, suggested a theory that deduces relations in accordance with Senarmont's findings. Following Senarmont, Stokes acknowledged the connection between the phenomena and the symmetry of the crystal, and employed it in his derivations. Like Lamé later, Stokes refrained from hypotheses about the particular structure of matter and the mechanism of heat transfer and preferred a formal reasoning. Yet he applied considerations of symmetry only in a specific case of two planes of symmetry. In that case he showed that the number of constants of heat conductivity reduces from nine to six. However, to reduce the number of constants in other cases (e.g., for the hexagonal system) he employed the crystalline form rather than its symmetry. In both cases the arguments were verbal and in this sense less formal than Lamé's. Although Lamé did not refer to Stokes in this connection it is likely that he was influenced by the latter. Still, mathematically they went in the opposite directions. Unlike Lamé, Stokes assumed symmetry in the crystal to deduce symmetry in the equations⁹. Stokes and Lamé made good use of symmetry in their mathematical studies. Yet they employed it sporadically and unsystematically and did not connect it methodically to the physical phenomena.

Conclusions

- 19 Lamé's use of symmetry differed from the exploitation of the concept in Haüy's tradition. Unlike their qualitative approach, Lamé employed the principle in a mathematical way. More importantly, unlike them he did it without much discussion of its physical meaning, or its connection with the experiment. Symmetry remained mainly a mathematical, rather than a physical, property in his science. In this vain asymmetry had no role at all in Lamé's work. He did not use symmetry or its lack to predict novel relations between phenomena, as others, most famously Pasteur did. It is tempting to connect this difference between Lamé and the French school to differences in their education. Haüy, Delafosse, Pasteur, Curie etc. were not graduates of the École polytechnique as Lamé was. The one important exception to this rule is Senarmont. The members of Haüy's school were not mathematical physicists, but crystallographers and chemists with a much closer connection with the experiment. Mathematical physicists in France at the time were more mathematicians with an interest in the methods of solving problems of physics than physicists seeking for means to answer issues in physics¹⁰. Indeed, like many of his mathematical physicist colleagues, Lamé was also an engineer. As other contributions to this volume show, part of his work in that field,

both as a practicing engineer and as a theoretician, was empirical especially at the earlier part of his career. Still, it seems that this empirical research did not influence Lamé's study of physics, which remained mathematical rather than empirical and, at least in the case of the analytical theory of heat, did not even explicitly relate its conclusions to specific physical conditions and conclusions.

- 20 In applying symmetry mathematically Lamé was close to the approach of Neumann's school. However, he shared neither their cautious attitude towards molecular assumption, nor the way by which they connected symmetry to physical properties. Of the early uses of symmetry his work seems to be most similar to that of his British colleague Stokes. Both were not closely connected with the French or the German school, although they were clearly influenced by the former through the work of Senarmont. Their influence on the later employment of symmetry was, at best, limited.
- 21 Senarmont's reference to symmetry was probably one of the factors that led Lamé to employ the concept, even if that did not ensure its employment as Duhamel's work exemplified. While a reconstruction of Lamé's reasons to employ considerations of symmetry can only be speculative, the issue that he studied surely formed a factor. Symmetry was considered in research on physical properties of crystals, like their heat conductivity. The major contributions and extensions of the rules of symmetry to physical phenomena were done by researchers who did not confine themselves to the structural-geometrical study of crystals. The number of related physical phenomena, however, was very small. At the middle of the nineteenth century considerations of symmetry were limited to elasticity, related issues of optics and heat conductivity, and in merely qualitative way to electric conductivity, all in crystals. That not many researchers studied these topics can explain the slow adoption of symmetry considerations. For some experimental physicists the mathematical or crystallographic concepts of symmetry were not well known or easy to use. On the other hand, in his research on the mathematical theory of crystals, Lamé acquired ease in employing the concept of symmetry. Lastly, the ability of employing symmetry to avoid particular hypotheses, even if not the only tool for this end, seems as an important reason for Lamé's use of the concept. Such an attitude was not the preference of every student of physics. Some preferred the employment of concrete assumptions about the physical structure of the matter at stake.

Major events in the application of symmetry to physics

1840	Delafosse's modification of Haüy's theory following considerations of symmetry
1847-50	Senarmont's experimental confirmation that heat and electric conductivity conform the symmetry of crystals
1851	Stokes's employment of symmetry in a theory of Senarmont's findings
1857	Pasteur's discovery of the organic agent in the fermentation that produces amy alcohol following considerations of symmetry
1861	Lamé's employment of symmetry in another theory of Senarmont's findings (F)
1873-74	Neumann's application of rotating coordinates in deriving the elastic constants of the holohedral crystals (F)
1876	Kirchhoff's derivation of the same constants using potential function (F)
1882	Voigt's extension of Neumann's and Kirchhoff's derivation to all crystals (F)
1882	Jacques and Pierre Curie's statement of conditions for the appearance of piezoelectricity in terms of crystal symmetry
1883	Aron's more elaborated derivation of the elastic constants in Kirchhoff's method (F)
1884	Minnigerode's yet more general derivation of the constants (F) and his rule for the symmetry of physical phenomena (D)
1884	Mallard's employment of consideration of symmetry to point out pyroelectric crystals
1884	Pierre Curie's study of symmetry including that of physical magnitudes (F)
1886	Minnigerode's derivation of the constants of heat conductivity (F)
1890	Voigt's derivation of the piezoelectric constants (D,F)
1894	Curie's rules for the application of symmetry to physical phenomena (Curie's principle) (D,F)

- 22 (D) - a discussion that includes an explicit rule relating symmetry and phenomena.
- 23 (F) - a formal mathematical treatment. 1832 Neumann's employment of symmetry to deduce elastic constants (F)

NOTES

1. For more details about this history see Katzir, Shaul, "The emergence of the principle of symmetry in physics", *Historical Studies in the Physical and Biological Sciences*, 35, 2004, pp. 35-66. The current article is based in parts on that paper.
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 7. Page numbers in parentheses refer to Lamé, Gabriel, *Leçons sur la théorie analytique de la chaleur*, Paris : Mallet-Bachelier, 1861.
 8. Lamé, Gabriel, *Examen des différentes méthodes employées pour résoudre les problèmes de géométrie*, Paris : Courcier, 1818.
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